**Physics 12** **Name:**

## Ultimate Circular Motion and Gravitation Assignment (16%)

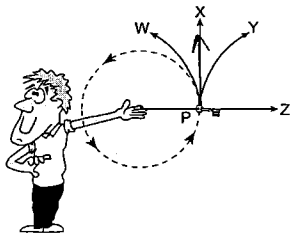
**Key Formulae:**

$$T = \frac{1}{f} \qquad a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} \qquad F = G\frac{m_1 m_2}{r^2} \qquad E_p = -G\frac{m_1 m_2}{r}$$

0108

1.

The diagram shows a student "twirling" a car key in a circular path on the end of a string.




If the string snaps at P, which path will the keys follow?

A. W
B. X
C. Y
D. Z

$\vec{v}$ is tangent to radius

2.

An athlete runs, at a constant speed, around a circle of radius 5.0 m in 12 s. What are the athlete's speed and acceleration?

| | SPEED | MAGNITUDE OF ACCELERATION |
|---|---|---|
| A. | 0.42 m/s ✗ | 0.22 m/s² ✗ |
| B. | 0.42 m/s ✗ | 1.4 m/s² ✓ |
| C. | 2.6 m/s ✓ | 0.22 m/s² ✗ |
| D. | 2.6 m/s ✓ | 1.4 m/s² ✓ |

$v = \frac{2\pi r}{T} = 2.6$ $\qquad a = \frac{4\pi^2 r}{T^2} = 1.4$

3.

A frictionless 3.0 kg cart rolls down an incline, and then "loops the loop."

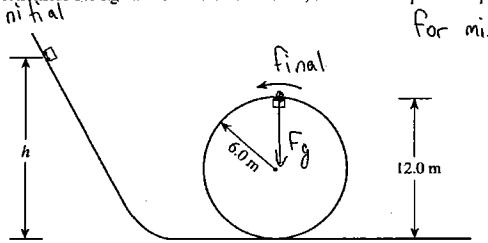



Diagram not to scale.

For minimum, $F_g = F_c$

$mg = \frac{mv^2}{r}$

$v^2 = gr$

Now, use energy!

From what minimum height, $h$, should the cart be released so that it does not fall off the circular track?

A. 12.0 m
B. 15.0 m
C. 18.0 m
D. 24.0 m

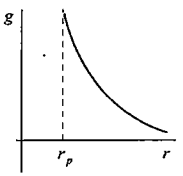
$PE_i + KE_i = PE_f + KE_f$

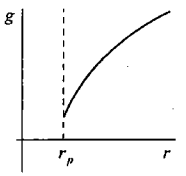
$mgh_i = mgh_f + \frac{1}{2}mv_f^2$

$h_i = \frac{gh_f + .5v_f^2}{g} = \frac{(9.8)(12) + .5(9.8)(6)}{9.8} = 15.0\text{ m}$

4.

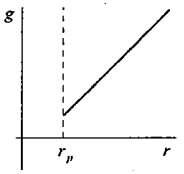
Which graph best shows how the gravitational field strength, $g$, varies with the distance, $r$, from the centre of a planet? ($r_p$ is the radius of the planet.)

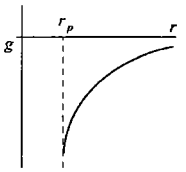
A.



B.



$g \propto \frac{1}{r^2}$

as $r \uparrow$, $g \downarrow$

C.



D.


5.

A satellite is in a stable circular orbit around the earth. Another satellite in a stable circular orbit at a greater altitude must have

$F_c = F_g$ $\quad \frac{m 4\pi^2 r}{T^2} = \frac{GMm}{r^2}$

$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$ as $r \uparrow$, $T \uparrow$

A. a smaller speed and a shorter period.
(B) a smaller speed and a longer period.
C. a greater speed and a shorter period.
D. a greater speed and a longer period.

also $\frac{mv^2}{r} = \frac{GMm}{r^2}$ $\quad v = \sqrt{\frac{GM}{r}}$ as $r \uparrow$, $v \downarrow$

6.

Which of the following could represent the kinetic energy, the gravitational potential energy and the total energy for an orbiting satellite in a stable circular orbit?

| | KINETIC ENERGY | GRAVITATIONAL POTENTIAL ENERGY | TOTAL ENERGY |
|---|---|---|---|
| (A.) | 40 000 J ✓ | −80 000 J ✓ | −40 000 J ✓ |
| B. | 40 000 J | 40 000 J | 80 000 J |
| C. | 80 000 J | 40 000 J | 120 000 J |
| D. | 80 000 J | −40 000 J | 40 000 J |

$KE = \frac{1}{2} PE$, but positive

$TE = KE + PE$

7.

A spacecraft of mass 470 kg rests on the surface of an asteroid of radius 1 400 m and mass $2.0 \times 10^{12}$ kg. How much energy must be expended so that the spacecraft may rise to a height of 2 800 m above the surface of the asteroid? **(7 marks)**

$W = \Delta PE = PE_f - PE_i$

$= -\frac{GM_1M_2}{r_f} - \frac{-GM_1M_2}{r_i}$

$m_2 = 470$

$M_1 = 2 \times 10^{12}$

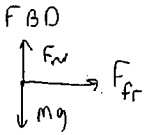
$r_i = 1400$

$r_f = 1400 + 2800 = 4200$

$= \frac{-(6.67 \times 10^{-11})(2 \times 10^{12})(470)}{4200} + \frac{(6.67 \times 10^{-11})(2 \times 10^{12})(470)}{1400}$

$= -14.93 + 44.78 = \boxed{29.9 \text{ J}}$

FBD

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8.

A car travels at 25 m/s along a horizontal curve of radius 450 m. What minimum coefficient of friction is necessary between its tires and the road in order for the car not to skid?

A. 0.14
B. 0.54
C. 0.72
D. 1.4

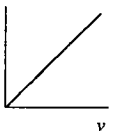
$F_{fr} = F_c$

$\mu mg = \frac{mv^2}{r}$

$\mu = \frac{v^2}{gr} = .142$

9.

In a series of test runs, a car travels around the same circular track at different velocities. Which graph best shows the relationship between its centripetal acceleration, $a_c$, and its velocity, $v$?

A. 


B. 


$a = \frac{v^2}{r}$

↑

$v^2$ means parabola

C. 


D. 


FBD


10.

Tarzan, of mass 85 kg, holds on to a horizontal vine of length 8.0 m and jumps off a cliff. What is the tension force in the vine as Tarzan passes the lowest point of his circular path?

A. 830 N
B. 1 700 N
C. 2 500 N
D. 6 700 N

$T - mg = F_c$

$T = mg + \frac{mv^2}{r}$

$PE_i = KE_f$ ← need $v_f$!

$mgh_i = \frac{1}{2}mv_f^2$

$v_f^2 = 2gh_i$ h = radius!

$v_f^2 = 2gr$

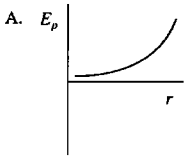
cool, huh?

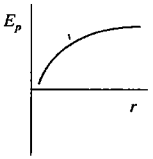
$T = mg + \frac{m2gr}{r}$

$T = 3mg = 2499$ N

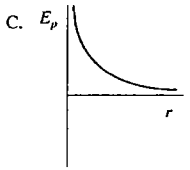
11.

Which graph shows gravitational potential energy plotted as a function of distance $r$ from the centre of the earth?

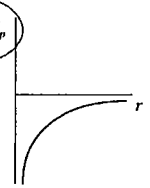
A. 


B. 


$PE = -\frac{GM_1m_2}{r}$

C. 


$PE \propto -\frac{1}{r}$

D. 


12.

How much work must be done to lift a $4.00 \times 10^4$ kg object from Earth's surface to a height of $3.00 \times 10^5$ m?

A. $1.12 \times 10^{11}$ J
B. $1.18 \times 10^{11}$ J
C. $2.39 \times 10^{12}$ J
D. $5.32 \times 10^{13}$ J

$W = \Delta PE = PE_f - PE_i$

$= -\frac{GM_1M_2}{r_f} - -\frac{GM_1M_2}{r_i}$

$r_f = 3 \times 10^5 + r_{earth}$

$= 6.68 \times 10^6$

$= \frac{(-6.67 \times 10^{-11})(5.98 \times 10^{24})(4 \times 10^4)}{6.68 \times 10^6} + \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(4 \times 10^4)}{6.38 \times 10^6}$

$= -2.388 \times 10^{12} + 2.501 \times 10^{12}$

$= 1.12 \times 10^{11}$ J

13. An 884 kg satellite in orbit around a planet has a gravitational potential energy of $-5.44 \times 10^{10}$ J. The orbital radius of the satellite is $8.52 \times 10^{6}$ m and its speed is $7.84 \times 10^{3}$ m/s.

a) What is the mass of the planet? **(3 marks)**

b) What is the kinetic energy of the satellite? **(2 marks)**

c) What is the total energy of the satellite? **(2 marks)**

a) $F_g = F_c$

$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow M = \frac{v^2 r}{G} = \frac{(7.84 \times 10^3)^2 (8.52 \times 10^6)}{6.67 \times 10^{-11}}$

$M_{planet} = 7.85 \times 10^{24}$ kg

b) $KE = \frac{1}{2}mv^2 = \frac{1}{2} PE$, but positive $= 2.72 \times 10^{10}$ J

c) $TE = PE + KE = -5.44 \times 10^{10} + 2.72 \times 10^{10}$

$= -2.72 \times 10^{10}$ J

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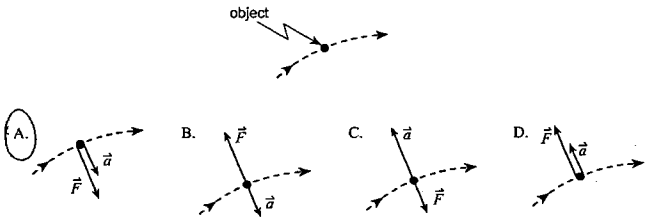
14.

Which vector diagram best represents the acceleration, $\vec{a}$, and force, $\vec{F}$, for an object travelling along a circular path?




15.

An object travels along a circular path with a constant speed $v$ when a force $F$ acts on it. How large a force is required for this object to travel along the same path at twice the speed ($2v$)?

A. $\frac{1}{2}F$

B. $F$

C. $2F$

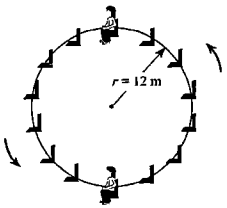
D. $4F$

$F = \frac{mv^2}{r}$ Now double speed $\Rightarrow \frac{m(2v)^2}{r}$

$= 4\frac{mv^2}{r} = 4 \times F$

16.

The diagram shows a 52 kg child riding on a Ferris wheel of radius 12 m and period 18 s. What force (normal force) does the seat exert on the child at the top and bottom of the ride?




| | TOP | BOTTOM |
|---|---|---|
| A. | 76 N ✗ | 76 N |
| B. | 430 N ✓ | 590 N |
| C. | 510 N ✗ | 510 N |
| D. | 590 N ✗ | 430 N |

Top

$F_N$

$mg$

$mg - F_N = F_c$

$F_N = mg - F_c$

$= (52)(9.8) - \frac{52(4)(\pi^2)(12)}{18^2}$

$= 430$ N

don't need to find bottom!

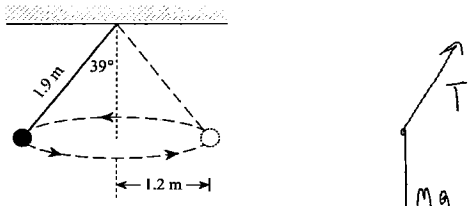
17.

The equation $E_p = mgh$, in which $g$ is 9.8 m/s$^2$, can not be used for calculating the gravitational potential energy of an orbiting Earth satellite because

A. the Earth is rotating.
B. of the influence of other astronomical bodies.
C. the Earth's gravity disappears above the atmosphere.
(D.) the Earth's gravitational field strength varies with distance.

18.

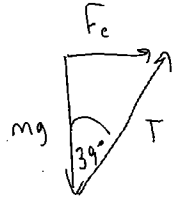
The diagram shows an object of mass 3.0 kg travelling in a circular path of radius 1.2 m while suspended by a piece of string of length 1.9 m. What is the centripetal force on the mass?







A. 19 N
B. 23 N
C. 24 N
D. 29 N




$\tan 39 = \frac{F_c}{mg}$

$F_c = mg \tan 39 = 23.8$ N

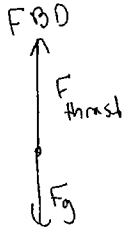
19.

a) Mars has a mass of $6.37 \times 10^{23}$ kg and a radius of $3.43 \times 10^6$ m. What is the gravitational field strength on its surface? **(4 marks)**

$g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(6.37 \times 10^{23})}{(3.43 \times 10^6)^2}$

$= 3.61$ N/kg

b) What thrust force must the rocket engine of a Martian lander exert if the 87.5 kg spacecraft is to accelerate upwards at 1.20 m/s$^2$ as it leaves the surface of Mars? **(3 marks)**




$F_T - F_g = ma$

$F_T = ma + mg$

$= (87.5)(1.2) + 87.5(3.61)$

$F_T = 421$ N

0008

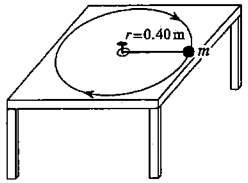
20.

A car travels at a uniform speed through a level circular curve in the road. Which of the following correctly describes the magnitude of the acceleration, velocity and force acting on the car?

| | MAGNITUDE OF ACCELERATION | MAGNITUDE OF VELOCITY | MAGNITUDE OF FORCE |
|---|---|---|---|
| A. | constant ✓ | constant ✓ | constant ✓ |
| B. | constant ✓ | changing ✓ | changing ✓ |
| C. | constant ✓ | changing ✓ | constant ✓ |
| D. | changing ✓ | changing ✓ | changing ✓ |

21.

An object is attached to a string that can withstand a maximum tension force of 6.3 N. The object travels in a circular path of radius 0.40 m with a period of 2.1 s.




$T_e = F_c$

$T_e = \frac{m 4\pi^2 r}{T^2}$

What is the maximum mass of the object?

A. 0.57 kg
B. 0.64 kg
C. 1.8 kg
D. 3.6 kg

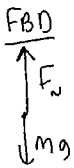
$m = \frac{T_e \cdot T^2}{4\pi^2 r} = \frac{(6.3)(2.1)^2}{4\pi^2 (.4)}$

22.

A 65 kg pilot in a stunt plane performs a vertical loop with a 700 m radius. The plane reaches a speed of 210 m/s at the bottom of the loop. What is the upward force on the pilot at the bottom of the loop?

A. 640 N
B. 3 500 N
C. 4 100 N
D. 4 700 N

FBD

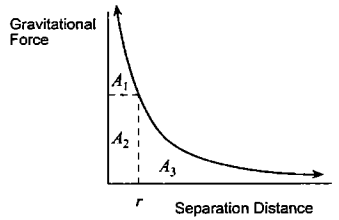



$F_N - mg = F_c$

$F_N = mg + \frac{mv^2}{r} = 4.73 \times 10^3$ N

23.

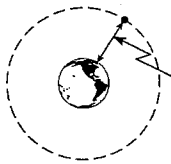
Which of the indicated areas of the graph represent the work needed to send an object from separation distance $r$ to infinity?




A. $A_1 + A_2$
B. $A_2$
C. $A_2 + A_3$
D. $A_3$

24.

A satellite experiences a gravitational force of 228 N at an altitude of $4.0 \times 10^7$ m above Earth.




$F = \frac{GMm}{r^2}$

What is the mass of this satellite?

A. 23 kg
B. 650 kg
C. 910 kg
D. 1 200 kg

$m = \frac{r^2 F}{GM} = \frac{(4\times10^7 + 6.38\times10^6)^2 (228)}{(6.67\times10^{-11})(5.98\times10^{24})}$

$m = 1230$ kg

25.

A 1 570 kg satellite orbits a planet in a circle of radius $5.94 \times 10^6$ m. Relative to zero at infinity the gravitational potential energy of this satellite is $-9.32 \times 10^{11}$ J. What is the mass of the planet?

A. $5.29 \times 10^{25}$ kg
B. $8.31 \times 10^{28}$ kg
C. $3.14 \times 10^{31}$ kg
D. $4.93 \times 10^{34}$ kg

$PE = \frac{-GMm}{r}$

$M = \frac{-r\,PE}{Gm} = \frac{-(5.94\times10^6)(-9.32\times10^{11})}{(6.67\times10^{-11})(1570)}$

$M = 5.3 \times 10^{25}$ kg