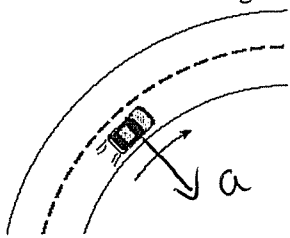


Circular Motion and Gravitation

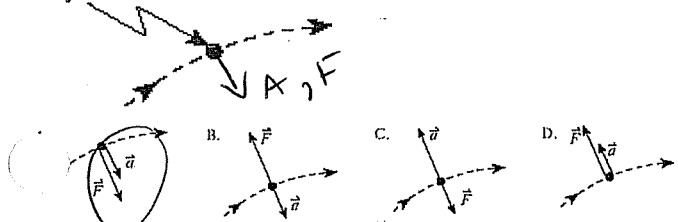
$v = \frac{d}{t} = \frac{2\pi r}{t} = 2.6 \text{ m/s}$ ,  $a = \frac{v^2}{r} = 1.4 \text{ m/s}^2$

1. A car is moving at a constant speed around a circular curve. Which of the following best describes this situation?



	VELOCITY OF CAR	ACCELERATION OF CAR	NET FORCE ON CAR
A.			
B.			
<b>C.</b>			
D.			

2. Which vector diagram best represents the acceleration,  $a$ , and force,  $F$ , for an object travelling along a circular path?

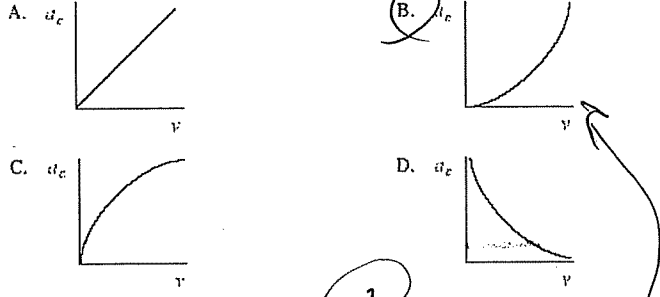


3. An object travels along a circular path with a constant speed  $v$  when a force  $F$  acts on it. How large a force is required for this object to travel along the same path at twice the speed ( $2v$ )?

A.  $12F$   
 B.  $F$   
 C.  $2F$   
**D.  $4F$**

$F = \frac{mv^2}{r}$      $F_{\text{new}} = \frac{m(2v)^2}{r} = 4 \frac{mv^2}{r} = 4F$

4. In a series of test runs, a car travels around the same circular track at different velocities. Which graph best shows the relationship between its centripetal acceleration,  $a_c$ , and its velocity,  $v$ ?



$a = \frac{v^2}{r}$  quadratic

5. An athlete runs, at a constant speed, around a circle of radius 5.0 m in 12 s. What are the athlete's speed and acceleration?

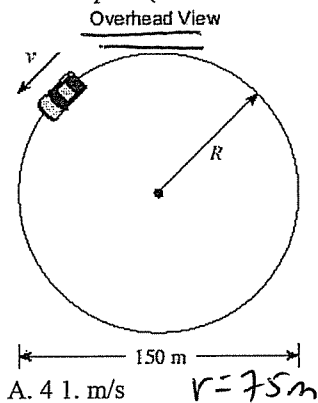
	SPEED	MAGNITUDE OF ACCELERATION
A.	0.42 m/s	0.22 m/s <sup>2</sup>
B.	0.42 m/s	1.4 m/s <sup>2</sup>
C.	2.6 m/s	0.22 m/s <sup>2</sup>
<b>D.</b>	2.6 m/s	1.4 m/s <sup>2</sup>

6. A car travels at 25 m/s along a horizontal curve of radius 450 m. What minimum coefficient of friction is necessary between its tires and the road in order for the car not to skid?

**A. 0.14**  
 B. 0.54  
 C. 0.72  
 D. 1.4

$F_f = F_c$      $\mu = \frac{v^2}{g \cdot r}$   
 $\mu mg = \frac{mv^2}{r}$

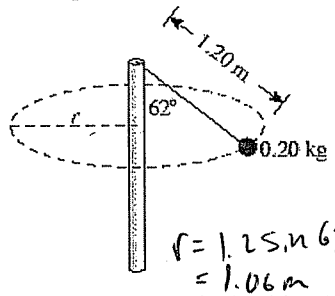
7. What is the maximum speed a car can travel along a level circular path (as shown below) if the coefficient of friction is 0.86?



- A. 41 m/s  
 B. 8.0 m/s  
**C. 25 m/s**  
 D. Depends on the mass of the car

$F_f = F_c$   
 $\mu mg = \frac{mv^2}{r}$   
 $v^2 = \mu g r$

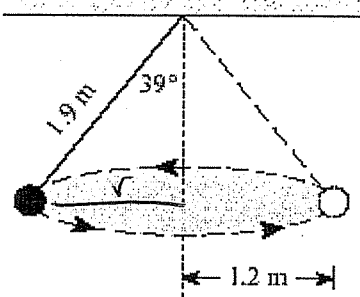
8. A 0.20 kg object moves at a constant speed in a horizontal circular path as shown.



$\tan 62^\circ = \frac{mv^2}{mg}$   
 $v^2 = g \cdot r \cdot \tan 62^\circ$

- What is the speed of this object?  
 A. 2.3 m/s  
 B. 3.2 m/s  
 C. 3.4 m/s  
**D. 4.4 m/s**

9. The diagram shows an object of mass 3.0 kg travelling in a circular path of radius 1.2 m while suspended by a piece of string of length 1.9 m. What is the centripetal force on the mass?



$$r = 1.9 \sin 39^\circ$$

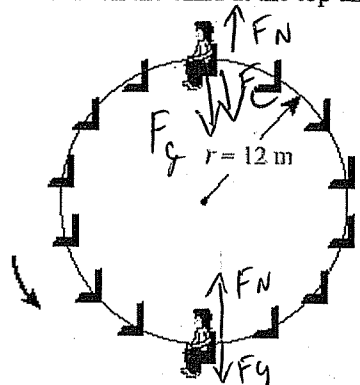


$$\tan 39^\circ = \frac{mv^2/r}{mg}$$

$$F_c = mg \tan 39^\circ$$

- A. 19 N
- B. 23 N
- C. 24 N**
- D. 29 N

10. The diagram shows a 52 kg child riding on a Ferris wheel of radius 12 m and period 18 s. What force (normal force) does the seat exert on the child at the top and bottom of the ride?



top  $F_{net} = F_g - F_N$

$$F_c = F_g - F_N$$

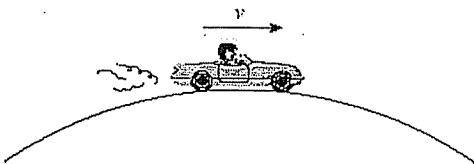
$$F_N = F_g - F_c$$

bottom  $F_c = F_N - F_g$

$$F_N = F_c + F_g$$

	TOP	BOTTOM
A.	76 N	76 N
<b>B.</b>	430 N	590 N
C.	510 N	510 N
D.	590 N	430 N

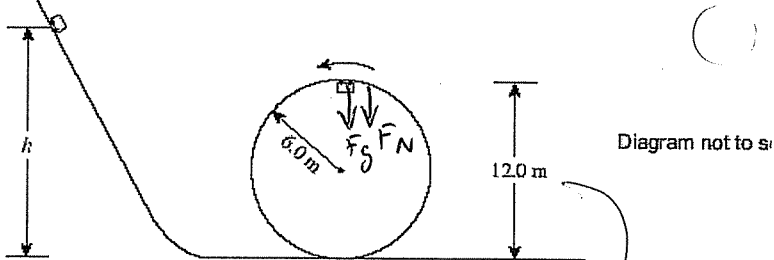
11. A vehicle and driver travel at constant speed over the hill as shown.



Which of the following free body diagrams best describes the vehicle at this position?

- A.**
- B.
- C.
- D.

12. A frictionless 3.0 kg cart rolls down an incline, and then "loops the loop."



From what minimum height,  $h$ , should the cart be released so that it does not fall off the track at the top?  $F_c = F_N + F_g$ ,  $F_N = 0$  for min  $h$ .

- A. 12.0 m
- B. 15.0 m**
- C. 18.0 m
- D. 24.0 m

$$F_c = F_g$$

$$\frac{mv^2}{r} = mg$$

$$v^2 = g \cdot r = (9.8)(6) = 58.8$$

$$h = 15 \text{ m}$$

13. Tarzan, of mass 85 kg, holds on to a horizontal vine of length 8.0 m and jumps off a cliff. What is the tension force in the vine as Tarzan passes the lowest point of his circular path?

- A. 830 N
- B. 1700 N
- C. 2500 N**
- D. 6700 N

$$mgh = \frac{1}{2}mv^2$$

$$F_{net} = T - F_g$$

$$\frac{mv^2}{r} = T - F_g$$

$$2mgh = T - F_g$$

14. Which of the following best illustrates how the gravitational field strength of a body varies with distance  $r$  from the body's centre?

- A.
- B.
- C.
- D.

$$g = \frac{GM}{r^2}$$

15. Which of the following graphs has a slope equal to the gravitational constant,  $G$ ?

- A.
- B.
- C.
- D.

$$F = G \frac{m_1 m_2}{r^2}$$

16. What is the gravitational force exerted on a 63 kg student by her 1400 kg car when their centers are 7.0 m apart?

- A.  $8.6 \times 10^{11}$  N
- B.  $1.9 \times 10^9$  N
- C.  $1.2 \times 10^7$  N
- D.  $1.8 \times 10^3$  N

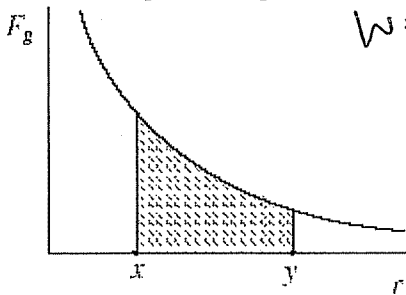
$$F = \frac{G M_1 m_2}{r^2}$$

17. A 75 kg astronaut stands on the surface of a planetoid with a mass of  $5.8 \times 10^{21}$  kg and a radius of  $7.3 \times 10^5$  m. What is the gravitational field strength at the surface?

- A. 0.73 N/kg
- B. 1.6 N/kg
- C. 9.8 N/kg
- D. 54 N/kg

$$g = \frac{G M_a}{r_a^2} = \frac{(6.67 \times 10^{-11})(5.8 \times 10^{21})}{(7.3 \times 10^5)^2}$$

18. As an object moves from x to y, the shaded area below the graph of gravitational force versus distance of separation represents



$$W = F \cdot d$$

- A. the gain in kinetic energy.
- B. the energy released into space.
- C. the work required to move the object.
- D. the average force required to move the object.

19. What is the change in gravitational potential energy as a 3500 kg object is raised vertically from the surface of the earth to a height of  $8.2 \times 10^5$  m?

- A.  $5.5 \times 10^7$  J
- B.  $2.5 \times 10^{10}$  J
- C.  $2.8 \times 10^{10}$  J
- D.  $1.9 \times 10^{11}$  J

$$E_p = G M m / r$$

$$\Delta E_p = -\frac{G M_e (3500)}{r_e + 8.2 \times 10^5} - \left( -\frac{G M_e (3500)}{r_e} \right)$$

20. How much work must be done to lift a  $4.00 \times 10^4$  kg object from Earth's surface to a height of  $3.00 \times 10^5$  m?

- A.  $1.12 \times 10^{11}$  J
- B.  $1.18 \times 10^{11}$  J
- C.  $2.39 \times 10^{12}$  J
- D.  $5.32 \times 10^{13}$  J

$$W = \Delta E = \Delta E_p$$

$$= -\frac{G M_e (4 \times 10^4)}{r_e + 3 \times 10^5} - \left( -\frac{G M_e (4 \times 10^4)}{r_e} \right)$$

21. What minimum energy is required to raise a  $1.7 \times 10^3$  kg vehicle from the surface of the Moon to a height of  $5.22 \times 10^6$  m?

- A.  $1.6 \times 10^9$  J
- B.  $3.6 \times 10^9$  J
- C.  $4.8 \times 10^9$  J
- D.  $1.4 \times 10^{10}$  J

$$\Delta E_p = -\frac{G M_m M_o}{r_m + 5.22 \times 10^6} - \left( -\frac{G M_m M_o}{r_m} \right)$$

$$F_g = F_c \text{ for orbit}$$

$$\frac{1}{2} \left( \frac{G M_1 m_2}{r} \right) = \frac{1}{2} m v^2$$

$$\frac{1}{2} - E_p = KE$$

$$\frac{G M_1 m_2}{r^2} = \frac{m v^2}{r}$$

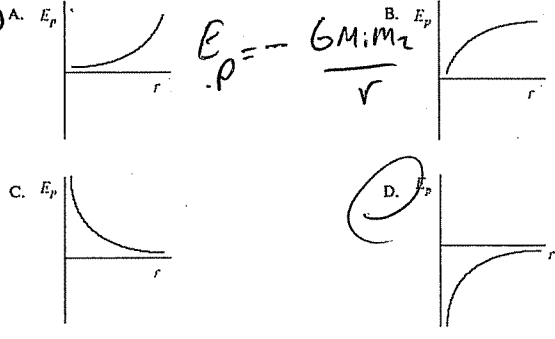
22. A 3500 kg piece of space debris is brought from an altitude of  $2.1 \times 10^5$  m back to the earth's surface. What is the change in potential energy of this space debris?

- A.  $-7.0 \times 10^9$  J
- B.  $-7.2 \times 10^9$  J
- C.  $-2.1 \times 10^{11}$  J
- D.  $-2.2 \times 10^{11}$  J

$$\Delta E_p = \dots \text{ Similar to 19}$$

$$= -\frac{G M_e M_o}{r_e} - \left( -\frac{G M_e M_o}{r_e + 2.1 \times 10^5} \right)$$

23. Which graph shows gravitational potential energy plotted as a function of distance r from the centre of the earth?



24. The equation  $E_p = mgh$ , in which  $g$  is  $9.8 \text{ m/s}^2$ , can not be used for calculating the gravitational potential energy of an orbiting Earth satellite because

- A. the Earth is rotating.
- B. of the influence of other astronomical bodies.
- C. the Earth's gravity disappears above the atmosphere.
- D. the Earth's gravitational field strength varies with distance.

25. A satellite is in a stable circular orbit around the earth. Another satellite in a stable circular orbit at a greater altitude must have

- A. a smaller speed and a shorter period.
- B. a smaller speed and a longer period.
- C. a greater speed and a shorter period.
- D. a greater speed and a longer period.

$$F_g = F_c \text{ less } F_g \text{ less } F_c$$

$$F_c = \frac{m v^2}{r} = \frac{m 4 \pi^2 r}{T^2}$$

26. A satellite orbits the earth with a speed of  $7.3 \times 10^3$  m/s. What is the distance from the centre of the earth to this satellite?

- A.  $2.3 \times 10^5$  m
- B.  $3.8 \times 10^6$  m
- C.  $7.5 \times 10^6$  m
- D.  $1.3 \times 10^7$  m

$$F_g = F_c$$

$$\frac{G M_e M_o}{r^2} = \frac{M_o v^2}{r}, r = \frac{G M_e}{v^2}$$

27. A  $2.0 \times 10^3$  kg satellite is in a circular orbit around the earth. The satellite has a speed of  $3.6 \times 10^3$  m/s at an orbital radius of  $3.1 \times 10^7$  m. What is the total energy of this orbiting satellite?

- A.  $-2.6 \times 10^{10}$  J
- B.  $-1.3 \times 10^{10}$  J
- C.  $-1.3 \times 10^{10}$  J
- D.  $-3.9 \times 10^{10}$  J

$$E_T = E_p + E_K$$

$$= -\frac{G M_1 M_2}{r} + \frac{1}{2} m v^2$$

28. Which of the following could represent the kinetic energy, the gravitational potential energy and the total energy for an orbiting satellite in a stable circular orbit?

	KINETIC ENERGY	GRAVITATIONAL POTENTIAL ENERGY	TOTAL ENERGY
<input checked="" type="radio"/> A.	40 000 J	-80 000 J	-40 000 J
B.	40 000 J	40 000 J	80 000 J
C.	80 000 J	40 000 J	120 000 J
D.	80 000 J	-40 000 J	40 000 J

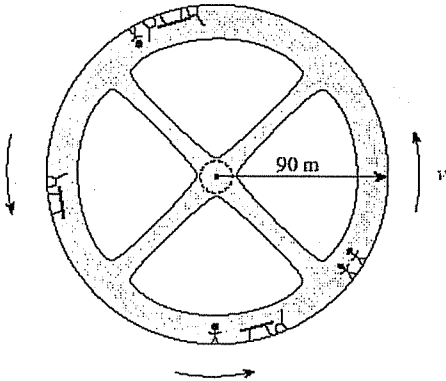
29. At an altitude of  $1.3 \times 10^7$  m above the surface of the earth an incoming meteor of mass  $1.0 \times 10^6$  kg has a speed of  $6.5 \times 10^3$  m/s. What would be the speed just before impact with the surface of the earth? Ignore air resistance.

- A.  $9.1 \times 10^3$  m/s
- B.  $1.0 \times 10^4$  m/s
- C.  $1.1 \times 10^4$  m/s
- D.  $1.7 \times 10^4$  m/s

$$E_T = E_K + E_P = E_K + E_P$$

$$\frac{1}{2} m_i v_i^2 + \left( \frac{-GM_e M_i}{r_e + 1.3 \times 10^7} \right) = \frac{1}{2} m_i v_f^2 + \left( \frac{-GM_e M_i}{r_e} \right)$$

30. A space station of radius 90 m is rotating to simulate a gravitational field.



a) What is the period of the space station's rotation so that a 70 kg astronaut will experience a normal force by the outer wall equal to 60% of his weight on the surface of the earth?

$$F_N = F_c = .6 (F_g) \quad T = 24.6 \text{ s}$$

$$\frac{m 4\pi^2 r}{T^2} = .6 m (9.8)$$

$$T^2 = \frac{4\pi^2 (90)}{(.6)(9.8)} = 604$$

b) What would be the effect experienced by the astronaut if the space station rotated faster so that the period of rotation was decreased? Explain your predicted effect.

Smaller  $T \rightarrow$  larger  $F_N$ , Feel heavier

31. A 720 kg communication satellite is in synchronous orbit around the planet Mars. This synchronous orbit matches the period of rotation so that the satellite appears to be stationary over a position on the equator of Mars. What is the orbital radius of this satellite?

**Planetary Data for Mars**

Mass:  $6.42 \times 10^{23}$  kg  
 Period of rotation:  $8.86 \times 10^4$  s

$$F_g = F_c$$

$$\frac{GM_m M_s}{r^2} = \frac{m 4\pi^2 r}{T^2}$$

$$r^3 = \frac{GM_m T^2}{4\pi^2} = 8.5 \times 10^{21}$$

$$r = 2.0 \times 10^7 \text{ m}$$

32. What minimum energy is required to take a stationary  $3.5 \times 10^3$  kg satellite from the surface of the Earth and put it into a circular orbit with a radius of  $6.88 \times 10^6$  m and an orbital speed of  $7.61 \times 10^3$  m/s? (Ignore Earth's rotation.)

$$W = \Delta E = E_f - E_o$$

$$= (E_K + E_P) - (E_P)$$

$$= \frac{1}{2} M_s v^2 + \left( \frac{-GM_e M_s}{6.88 \times 10^6} \right) - \left( \frac{-GM_e M_s}{r_e} \right)$$

$$= 1.01 \times 10^{10} \text{ J} - 2.03 \times 10^{10} \text{ J} + 2.19 \times 10^{10} \text{ J}$$

$$= 1.17 \times 10^{10} \text{ J}$$

33. An 884 kg satellite in orbit around a planet has a gravitational potential energy of  $5.44 \times 10^{10}$  J. The orbital radius of the satellite is  $8.52 \times 10^6$  m and its speed is  $7.84 \times 10^3$  m/s.

What is the mass of the planet?

$$E_p = -\frac{G M_p M_o}{r}$$

$$r = \frac{-G(8.52 \times 10^6)(884)}{-5.44 \times 10^{10}}$$

$$M_p = \frac{E_p \cdot r}{G M_o}$$

$$M_p = 7.86 \times 10^{24}$$

b) What is the kinetic energy of the satellite?

$$\frac{1}{2} M_s v^2 = 2.7 \times 10^{10} \text{ J}$$

34. A spacecraft of mass 470 kg rests on the surface of an asteroid of radius 1400 m and mass  $2.0 \times 10^{12}$  kg. How much energy must be expended so that the spacecraft may rise to a height of 2800 m above the surface of the asteroid?

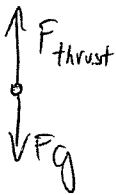
$$\Delta E_p = E_f - E_o$$

$$= -\frac{G M_a M_s}{4200 \text{ m}} - \left( -\frac{G M_a M_s}{1400 \text{ m}} \right)$$

a) Mars has a mass of  $6.37 \times 10^{23}$  kg and a radius of  $3.43 \times 10^6$  m. What is the gravitational field strength on its surface?

$$g = \frac{G M_p}{r_p^2} = 3.61 \text{ N/kg}$$

b) What thrust force must the rocket engine of a Martian lander exert if the 87.5 kg spacecraft is to accelerate upwards at  $1.20 \text{ m/s}^2$  as it leaves the surface of Mars?



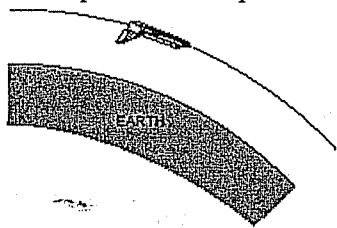
$$F_{\text{net}} = F_T - F_g$$

$$F_T = F_{\text{net}} + F_g$$

$$= (87.5)(1.2) + (87.5)(3.61)$$

$$= 421 \text{ N}$$

36. A space shuttle is placed in a circular orbit at an altitude of  $3.00 \times 10^5$  m above Earth's surface.



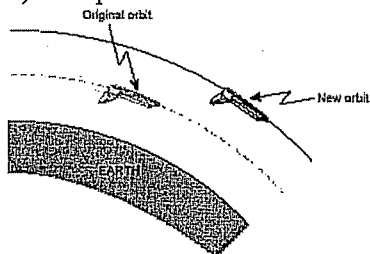
a) What is the shuttle's orbital speed?

$$F_g = F_c$$

$$\frac{G M_e M_s}{(r_e + 3 \times 10^5)^2} = \frac{M_s v^2}{(r_e + 3 \times 10^5)}$$

$$v^2 = \frac{G M_e}{6.68 \times 10^6 \text{ m}}$$

b) The space shuttle is then moved to a higher orbit in order to capture a satellite.



The shuttle's speed in this new higher orbit will have to be

- greater than in the lower orbit.
- less than in the lower orbit.
- the same as in the lower orbit.

c) Using principles of physics, explain your answer to b).

Velocity is proportional to  $\frac{1}{\sqrt{r}}$

37. A 5.0 kg rock dropped near the surface of Mars reaches a speed of 15 m/s in 4.0 s.

a) What is the acceleration due to gravity near the surface of Mars?

$$v_f = v_0 + at$$

$$15 = 0 + a(4)$$

$$a = \frac{15}{4} = 3.75 \text{ m/s}^2$$

b) Mars has an average radius of  $3.38 \times 10^6$  m. What is the mass of Mars?

$$g = \frac{G M_m}{r^2} = 3.75 \text{ N/kg}$$

$$M_m = \frac{(3.75)(3.38 \times 10^6)^2}{G} = 6.42 \times 10^{23} \text{ kg}$$

Answers:

- |      |       |       |   |  |  |
|------|-------|-------|---|--|--|
| 1. C | 9. C  | 17. A | 25. B   | 31. $2.0 \times 10^7$ m                              | 36. $7.73 \times 10^3$ m/s, less than..., v proportional to $\sqrt{r}$ |
| 2. A | 10. B | 18. C | 26. C   | 32. $1.17 \times 10^{11}$ J                          | 37. $3.75 \text{ m/s}^2$ , $6.42 \times 10^{23}$ kg                    |
| 3. D | 11. A | 19. B | 27. B   | 33. $7.86 \times 10^{24}$ kg, $2.7 \times 10^{10}$ J |  |
| 4. B | 12. B | 20. A | 28. A   | 34. 30 J   |  |
| 5. D | 13. C | 21. B | 29. C   | 35. 3.61 N/kg, 421 N                                 |  |
| 6. A | 14. D | 22. A | 30. 24.6 s, faster, less T, more $F_c$ , feel heavier |  |  |
| 7. C | 15. C | 23. D |   |  |  |
| 8. D | 16. C | 24. D |   |  |  |