

Gravitation

$1.98 \times 10^{30} \text{ kg}$

1. The gravitational force of attraction between the Sun and an asteroid travelling in an orbit of radius  $4.14 \times 10^{11} \text{ m}$  is  $4.62 \times 10^{17} \text{ N}$ . What is the mass of the asteroid?

$$F_g = \frac{G m_1 m_2}{r^2}, \quad 4.62 \times 10^{17} \text{ N} = \frac{(6.67 \times 10^{-11}) (1.98 \times 10^{30} \text{ kg}) m_2}{(4.14 \times 10^{11} \text{ m})^2}$$


$$m_2 = 6.0 \times 10^{20} \text{ kg}$$

2. A certain planet has a mass of  $3.3 \times 10^{23} \text{ kg}$  and a radius of  $2.6 \times 10^6 \text{ m}$ . What is the acceleration due to gravity on the surface of this planet?

$$a = g = \frac{G M P}{r^2} = \frac{(6.67 \times 10^{-11}) (3.3 \times 10^{23} \text{ kg})}{(2.6 \times 10^6 \text{ m})^2} = 3.3 \text{ m/s}^2$$

3. A satellite travels in a circular orbit at a height of one Earth radius above the surface of the Earth. What is the satellite's orbital period?


$r = 2r_e = 1.28 \times 10^7 \text{ m}$



$F_{\text{net}} = F_g$   
 $F_c = F_g$   
 $\frac{m 4\pi^2 r}{T^2} = \frac{G m_1 m_2}{r^2}$

$T^2 = \frac{4\pi^2 r^3}{G M_e}, \quad T = 1.4 \times 10^4 \text{ s}$

4. A satellite orbits a planet of mass  $4.0 \times 10^{25} \text{ kg}$  at a velocity of  $5.8 \times 10^3 \text{ m/s}$ . What is the radius of this orbit?



$F_{\text{net}} = F_g$   
 $F_c = F_g$   
 $\frac{m v^2}{r} = \frac{G m_1 m_2}{r^2}$

$r = \frac{G M P}{v^2} = 7.9 \times 10^7 \text{ m}$

**Assignment: Day 1**

5. Find the gravitational force of attraction between a 75 kg physics student and her 1 500 kg car when their centres are 10 m apart.

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11}) (75) (1000)}{10^2} = 7.5 \times 10^{-8} \text{ N}$$

6. A satellite is travelling around the Earth in an orbit of radius  $4.47 \times 10^7 \text{ m}$ . What is the mass of the satellite if it experiences a gravitational force of  $3.00 \times 10^3 \text{ N}$ ?

$$F_g = \frac{G m_1 m_2}{r^2}, \quad 3.0 \times 10^3 = \frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24}) (M_{\text{sat}})}{(4.47 \times 10^7)^2}$$

$$M_{\text{sat}} = 1.5 \times 10^4 \text{ kg}$$

7. An astronaut stands on the surface of a planet of radius  $2.6 \times 10^6 \text{ m}$ . An object dropped from the astronaut's hand accelerates at  $3.2 \text{ m/s}^2$ .

a) What is the mass of this planet?

$$a_c = g = \frac{GM_p}{r^2}, \quad 3.2 \text{ m/s}^2 = \frac{(6.67 \times 10^{-11}) M_p}{(2.6 \times 10^6)^2}, \quad M_p = 3.2 \times 10^{23} \text{ kg}$$

b) What is the force of gravity on an 18 kg mass located on the surface of this planet?

$$F_g = mg = (18 \text{ kg})(3.2 \text{ m/s}^2) = 58 \text{ N}$$

8. What is the gravitational field strength at the surface of a star of mass  $4.8 \times 10^{31} \text{ kg}$  and radius  $2.7 \times 10^8 \text{ m}$ ?

$$g = \frac{GM_p}{r^2}, \quad \frac{(6.67 \times 10^{-11})(4.8 \times 10^{31} \text{ kg})}{(2.7 \times 10^8 \text{ m})^2} = 4.4 \times 10^4 \text{ N/kg}$$

9. A space shuttle orbits the earth at an altitude where the acceleration due to gravity is  $8.70 \text{ m/s}^2$ . What is the shuttle's speed at this altitude?

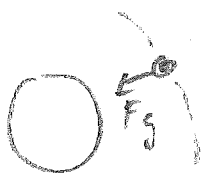
$$g = \frac{GM_e}{r^2}, \quad 8.7 = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{r^2}, \quad r = 6.77 \times 10^6 \text{ m}$$

$$a_c = g = \frac{v^2}{r}, \quad 8.7 = \frac{v^2}{6.77 \times 10^6}, \quad v = 7.68 \times 10^3 \text{ m/s}$$

10. A circular space station of radius 120 m is to be rotated so that its astronauts experience an effect similar to that of a gravitational field. If the field is to be  $5.0 \text{ m/s}^2$  at this radius, what should be the period of rotation of the space station?

$$a_c = g, \quad \frac{4\pi^2 r}{T^2} = 5.0 \text{ m/s}^2, \quad T^2 = \frac{4\pi^2 (120)}{5}, \quad T = 3.1 \times 10^1 \text{ s}$$

11. A 1500 kg spaceship circles a planet once every  $4.0 \times 10^5 \text{ s}$  with an orbital radius of  $3.6 \times 10^7 \text{ m}$ . What is the mass of this planet?



$$F_{\text{net}} = F_g, \quad F_c = F_g, \quad M_p = \frac{4\pi^2 r^3}{G T^2} = 1.7 \times 10^{23} \text{ kg}$$

$$\frac{m 4\pi^2 r}{T^2} = \frac{G m_p m}{r^2}$$

**Enrichment: Day 1**

12. a) The space shuttle orbits the Earth in a circular path where the gravitational field strength is  $8.68 \text{ N/kg}$ . What is the shuttle's orbital radius?

$$g = \frac{GM_p}{r^2}, \quad 8.68 \text{ N/kg} = \frac{G(5.98 \times 10^{24} \text{ kg})}{r^2}$$

b) A space station that has 10 times the mass of the shuttle in a) orbits Earth at the same altitude.

How does the orbital speed of the space station compare to that of the shuttle? (Check one response.)

- The space station's speed is less than the shuttle's speed.
- The space station's speed is the same as the shuttle's speed.
- The space station's speed is greater than the shuttle's speed.

c) Using principles of physics, explain your answer to b).

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMEm}{r^2}, \quad v^2 = \frac{GM_e}{r}, \quad v \text{ only based on } r'$$

13. A satellite is placed in circular orbit at an altitude of  $4.8 \times 10^5$  m above Earth's surface. What is the satellite's orbital period?

$$r = r_e + 4.8 \times 10^5 \text{ m} = 6.86 \times 10^6 \text{ m}$$

$$F_c = F_g$$

$$\frac{4\pi^2 r}{T^2} = \frac{GM_e m}{r^2}, \quad T^2 = \frac{4\pi^2 r^3}{GM_e}$$

b) (i) As shown in the diagram below, two satellites pass over the same point on Earth's surface. Satellite H is in a higher orbit than satellite L.

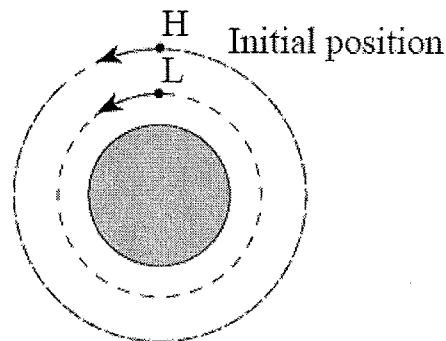
Which satellite, H or L, completes one orbit first? (Circle one)

A. satellite H

B. satellite L

(ii) Using principles of physics, explain your answer.

$$T^2 = \frac{4\pi^2 r^3}{GM_e} \quad \text{smaller } r \text{ produces smaller } T$$



14. A planet of radius  $7.0 \times 10^7$  m has a gravitational field strength of 68 N/kg at its surface. What is the period of a satellite orbiting this planet at a radius of  $1.4 \times 10^8$  m (twice the planet's radius)?

$$g = \frac{GM_p}{r^2}, \quad M_p = \frac{g r^2}{G} = 5 \times 10^{27} \text{ kg}$$

$$F_c = F_g$$

$$\frac{m 4\pi^2 r}{T^2} = \frac{GM_p m}{r^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_p}$$

15. The moon Deimos orbits the planet Mars at an orbital radius of  $2.34 \times 10^7$  m with an orbital period of  $1.08 \times 10^5$  s. What is the mass of Mars?

$$F_c = F_g$$

$$m \frac{4\pi^2 r}{T^2} = \frac{GMm}{r^2}, \quad M = \frac{4\pi^2 r^3}{T^2 G}$$

Assignment: Day 2

16. A satellite orbits the Earth with a speed of  $5.2 \times 10^3$  m/s. What is the satellite's distance from the centre of the Earth?

$$F_c = F_g$$

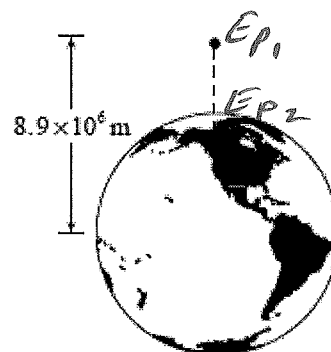
$$\frac{mv^2}{r} = \frac{GM_1M_2}{r^2} \quad , \quad r = \frac{GM_e}{v^2}$$

17. A stationary 25 kg object is released from a position  $8.9 \times 10^6$  m from the centre of the earth. What is the speed of the object just before impact? Ignore air resistance.

$$E_b = E_a$$

$$E_{p1} = E_{p2} + E_k$$

$$-\frac{GM_em_s}{8.9 \times 10^6 \text{ m}} = -\frac{GM_em_s}{6.38 \times 10^6} + \frac{1}{2}m_s v^2$$

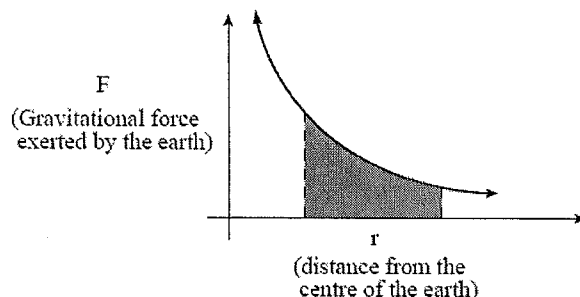


18. A 900 kg satellite which is travelling at 8 600 m/s around a planet of mass  $8.1 \times 10^{25}$  kg has an orbital radius of  $7.3 \times 10^7$  m. What is the total orbital energy of this satellite relative to infinity?

$$E_T = E_K + E_P$$

$$= \frac{1}{2}M_s v^2 + \frac{-GM_p M_s}{r}$$

19. The shaded area shown in the diagram represents
- A. the gravitational field strength near the earth.
  - B. the escape velocity from the surface of the earth.
  - C. the centripetal acceleration of an object orbiting the earth.
  - D. the work required to move an object in the earth's gravitational field.



20. A 1 250 kg rocket rests on the surface of the Earth. To what maximum distance from the Earth's centre would the rocket be lifted if  $2.5 \times 10^{10}$  J of work were done on it?

$$W = \Delta E = E_{pf} - E_{p0}$$

$$2.5 \times 10^{10} \text{ J} = -\frac{GM_e M_s}{r_f} - \left(-\frac{GM_e M_s}{r_e}\right)$$

21. An object is located on the surface of a planet. The **work** required to remove this object from the planet's gravitational field depends on which combination of the following three variables: mass of the planet, mass of the object, and radius of the planet?

	MASS OF PLANET	MASS OF OBJECT	RADIUS OF PLANET
<u>A.</u>	Yes ✓	Yes ✓	Yes ✓
B.	Yes	Yes	No
C.	Yes	No	Yes
D.	No	Yes	Yes

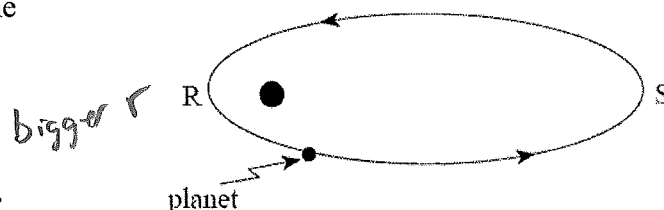
**Enrichment: Day 2**

22. Relative to zero at infinity, what is the gravitational potential energy of a  $7.2 \times 10^2$  kg satellite that is at a distance of  $3.4 \times 10^7$  m from earth's centre?

$$E_p = - \frac{G M_1 m_2}{r^2}$$

23. A planet is in orbit as shown in the diagram below. The planet's gravitational potential energy will

- A. be constant throughout its orbit.  
 B. always be equal to its kinetic energy.  
 C. increase as the planet goes from point R to point S.  
 D. decrease as the planet goes from point R to point S.



24. A 120 kg astronaut stands on the surface of an asteroid of radius 600 m. The astronaut leaves the surface with 15 J of kinetic energy and reaches a maximum height of 300 m above the surface. What is the mass of the asteroid?

$$W = \Delta E = E_{Pf} - E_{Po}$$

$$15J = - \frac{GM_a(120)}{900} - \frac{GM_a(120)}{300}$$

*factor out  $M_a$*

25. The work required to move an object in a planet's gravitational field can be determined graphically by calculating

- A. the slope of a graph of gravitational force versus separation distance.  
 B. the area under a graph of gravitational force versus separation distance.  
 C. the slope of a graph of gravitational potential energy versus separation distance.  
 D. the area under a graph of gravitational potential energy versus separation distance.

**Answers:**

- |                                  |                                 |                             |
|----------------------------------|---------------------------------|-----------------------------|
| 1. $6.0 \times 10^{20}$ kg       | 12. $6.78 \times 10^6$ m, same, | 23. D                       |
| 2. $3.3 \text{ m/s}^2$           | 13. $5.7 \times 10^3$ s, b) L,  | 24. $3.4 \times 10^{12}$ kg |
| 3. $1.4 \times 10^4$ s           | 14. $1.3 \times 10^4$ s         | 25. B                       |
| 4. $7.9 \times 10^7$ m           | 15. $6.5 \times 10^{23}$ kg,    |                             |
| 5. $7.5 \times 10^{-8}$ N        | 16. $1.5 \times 10^7$ m         |                             |
| 6. $1.50 \times 10^4$ kg         | 17. $6.0 \times 10^3$ m/s       |                             |
| 7. $3.2 \times 10^{23}$ kg, 58N, | 18. $-3.3 \times 10^{10}$ J     |                             |
| 8. $4.4 \times 10^4$ N/kg        | 19. D                           |                             |
| 9. $7.68 \times 10^3$ m/s        | 20. $9.39 \times 10^6$ m        |                             |
| 10. $3.1 \times 10^1$ s          | 21. A                           |                             |
| 11. $7 \times 10^{23}$ kg        | 22. $-8.4 \times 10^9$ J        |                             |